QUERIES AND INFORMATION.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

Why was the letter π chosen to represent $\frac{c}{d}$?

LOTTIE SMITH, Houston, Miss.

I have several problems, which though solved by quadratics, have one positive root and no other. I have also several problems, which can only be solved by quadratics, which have two roots, one real and one imaginary, not withstanding that "imaginary expressions enter an equation by pairs," which at present I will not disclose. The following problem is from Bell's Algebra (Chamber's Edition Course): "Given $\sqrt{2x^2-2})=3x-5$. $x=3$, or $\frac{9}{7}$." The $\frac{9}{7}$ fails to verify the equation. Can another root besides 3 be found that will?

R. GREENWOOD, Morris, Ill.

COMMENTS ON PROBLEM 11—GEOMETRY.

What does the gentleman do with the parts of the circle outside of his own central circle and the seven circles he gives to his seven children? If he does not "retain" it, he must think that these pieces will suit his wife.

W. F. BRADBURY,
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When the condition of the problem is satisfied, one of the seven equal circular farms will be concentric with the original farm. This condition is, therefore, incompatible with the (insinuated) condition that the gentleman shall retain for himself an area about the center of the original farm. The "problem" is merely a puzzle.

L. E. PRATT, Tecumseh, Neb.

On page 249 of Wentworth's *College Algebra*, we find the author conclude that $\underline{|0}=1$, *i. e.*, factorial zero is equal to unity. On page 246 the definition of *factorial* is given: $\underline{|n}=n(n-1)(n-2)\ldots\ldots1$, *i. e.*, factorial n is equal to the product of all the natural numbers from n to 1 inclusive. If n were 3, we would have $\underline{|3}=3.2.1$; if $n=8$, then $\underline{|8}=8.7.6\ldots\ldots1$. So for any other number. If therefore 0 is to be one of them, it must submit to the same law.

$\therefore$ $\underline{|0}=0\ldots\ldots1=0.1=0$! This would show that factorial zero, if it has any meaning at all, must be equal to zero. But *factorial zero* is not comprised within the definition given by the author: by that definition the first factor of the product is the number given (n), the last is unity, (1), which therefore excludes 0. The mistake made by the author in arriving at the result consists in disregarding the factor 0 in one of the terms of a fraction. From the formula

$C_{n,r}=\dfrac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{\underline{|r}}$ is derived, by multiplying each

term by $\underline{|n-r}$, the formula : $C_{n,r}=\dfrac{n(n-1)\ldots\ldots(n-r+1)(n-r)\ldots\ldots1}{\underline{|r}\times(n-r)\ldots\ldots1}\ldots[A]$,

or $C_{n,r}=\dfrac{\underline{|n}}{\underline{|r}\ \underline{|n-r}}$ Now if in this latter we make $n=r=1$ we obtain

$$C_{1,1}=\frac{\underline{|1}}{\underline{|1}\ \underline{|1-1}}=\frac{1}{1.\underline{|0}}=\frac{1}{\underline{|0}}\ldots\ldots(B).$$

[But as the number of combinations made of one element with one in the group is one, we also have

$$C_{1,1}=1.\quad \therefore 1=\frac{1}{\underline{|0}}\quad \therefore\ \underline{|0}=1.]$$

But suppose we make the substitution in formula [A] ;

$$C_{n,r}=\frac{n(n-1)\ldots\ldots(n-r+1)(n-r)\ldots\ldots1}{\underline{|r}\,(n-r)\ldots\ldots1}$$

where $(n-r+1)=(1-1+1)=1$.

$$C_{1,1}=\frac{(1)(n-r)\ldots\ldots1}{\underline{|1}\,(n-r)\ldots.1}=\frac{1}{1}.\frac{\underline{|n-r}}{\underline{|n-r}}=\frac{1}{1}=1 \text{ and not:} \frac{1}{\underline{|0}}.$$

The factor which becomes 0 in the denominator also occurs in the numerator and is to be cancelled. The error made as in [B], consists in neglecting this factor in the numerator but retaining it in the denominator.

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ANSWERS TO QUERIES IN MONTHLY FOR MARCH, 1894, (VOL. 1, NO. 3, P. 103.)

By Prof. John N. Lyle, Fulton, Mo.

I. Whether Lobatschewsky's theorem 4 is "sound" or not depends upon what shall be regarded as "sound" in geometry. If the assumption that a plane is the surface of a sphere and that two straight lines drawn therein perpendicular to a third do intersect is sound; then Lobatschewsky's theorem 4, since it contradicts this assumption, must be unsound. Otherwise, two propositions that contradict each other may both of them be sound. Again, if the soundness of Euclid's propositions 27 and 28, Book 1. is granted, that of Lobatschewsky's theorem 4 must also be conceded, since it is a legitimate corollary of those propositions.

II. Lobatschewsky's theorem 4 which reads as follows : "Two straight

lines perpendicular to a third never intersect, how far soever they be produced" contradicts flatly the assumption that these perpendiculars do intersect, no matter where the intersection is supposed to occur. The *fact* and not the *place* of supposed intersection constitutes the contradiction. Von Staudt's assumption that two straight lines perpendicular to a third have "at infinity a common point" contradicts proposition 27, Book 1. of Euclid's Elements, and hence can not be in harmony with it. Euclidean space cannot be extended to any point of intersection of the two perpendiculars under notice for the good and sufficient reason that those perpendiculars do not and can not intersect in that space.

III. No. IV. Yes.

V. No, for the reason that it involves contradiction. By definition every straight line having two ends is finite. Hence, to affirm that such a line is infinite in length is to attribute to it contradictory attributes. No infinite straight line can be drawn between two points located in space and geometrical science does not concern itself with what is supposed to occur or not to occur outside of space. Juggling with algebraical symbols can not alter the cold, hard facts of the Euclidean geometry.

VI. In his theorem 16 Lobatschewsky is *studiously silent* as to whether he regards the boundary line itself as a *cutting* or a *not-cutting* line. In his theorem 33, however, he uses this language—"hence not only does the distance between two parallels decrease (Theorem 24), but with the prolongation of the parallels towards the side of the parallelism this at last wholly vanishes. Parallel lines have therefore the character of asymptotes." From this it appears that Lobatschewsky holds that the distance between asymptotes and their curves "at last wholly vanishes."

VII. In theorems 32 and 33 Lobatschewsky exhibits without disguise his use and interpretation of the symbols 0 and ∞, and his speculative opinions respecting geometrical data that dominate his thinking and thus determine his conclusions. The reason assigned by Lobatschewsky for his conclusion that the distance between parallels decreases and "at last wholly vanishes" is that $s'=0$ for $x=\infty$ in the formula $s'=se^{-x}$. There is nothing novel, brilliant or profound in manipulating algebraical symbols in such fashion. It is in fact a familiar game of analytical sophistry more than two hundred years old played in the school of Leibnitz with 0 and ∞ as dice. In his theorem 32 Lobatschewsky informs us that "A circle with continually increasing radius merges in the boundary line." He further says that "one may also call the boundary line *a circle with infinitely great radius.*"

When Lobatschewsky rejected Euclid's axiom 12 and accepted in its stead a straight line as the circumference he evidently strained at a gnat and swallowed a camel. In Lotze's Metaphysics, Part II., Vol. I., pages 290 and 291 the fol-

lowing extract is found: "A finite arc of a circle, of course, becomes perpetually more like a straight line as the radius of the circle to which it belongs is increased; but the whole circle never comes to be like one. However infinitely great we may conceive the radius as being, nothing can prevent us from conceiving it to complete its rotation around the center, and till such rotation is completed we have no right to apply the conception of a circle to the figure which is generated: discourse about a straight line which being in secret a circle of infinite diameter, returned into itself, is not a portion of esoteric science, but a proof of logical barbarism. Just the same is shown by phrases about parallel lines which are supposed to cut each other at an infinite distance. They do not cut each other at any finite distance, and as every distance when conceived as attained would become finite again, there is simply no distance at which they do so; it is utterly inadmissible to pervert this negation into the positive assertion that in infinite distance there is a point at which intersection occurs."

EDITORIALS.

We were compelled to omit the Department of Geometry in this issue because of lack of sorts, and the Miscellaneous Department because this number has now grown far beyond its proper limits.

No pains will be spared on the part of the editors to make Vol. III. of great value to all its readers. To this end, we trust that we may have the coöperation of all of our old contributors and that of many new ones.

Professor E. L. Sherwood should have been given credit for solving Problem 46, Department of Geometry. Editor Colaw and Prof. Cooper D. Schmitt each sent a solution of Problem 54, Department of Arithmetic, but too late for credit in the proper place.

A correspondent who has a large collection of mathematical autographs and MSS. will exchange duplicates with any other who is interested in the same line. Professor Finkel will put this correspondent in communication with any one who will send his address.

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